Handout for Week 7: Logic II

Philosophy of Language.

Metavocabularies of Reason:

Pragmatics, Semantics, and Logic

https://sites.pitt.edu/~rbrandom/Courses

1. Two related senses of **implicit/explicit** in *reason relations* and *deontic pragmatics*:

First Parallel:

a1) **Γ|~A**: G∈Γ is part of the *explicit* **content** of Γ (is explicitly contained in Γ, as a member of that set), and A is part of the *implicit* content of Γ (in the literal sense of being *implied by* Γ).

a2) **Γ|~A**: the *explicit* **commitment** to *accept* all of Γ *precludes entitlement* to *deny* A, which is *implicit* commitment to *accept* A—in the sense that acceptance is the only sort of attitude (to accept/reject) to which one can be *entitled* to commit oneself w/res to A, since Γ|~A.

Second Parallel:

b1) **Explicitation**: when Γ|~A, we make the implicit content A explicit as a premise and compare the implicit content of Γ (its consequences) with the implicit content of $\Gamma\cup\{A\}$, i.e. {X: Γ|~X} and {Y: Γ,A|~Y}.

CM entails {X: Γ|~X}⊆{Y: Γ,A|~Y}. Explicitation does not *subtract* consequences.

CT entails {Y: Γ,A|~Y}⊆{X: Γ|~X}. Explicitation does not *add* consequences.

Together they entail that *explicitation is inconsequential*: {Y: Γ,A|~Y}={X: Γ|~X}.

b2) **Inference** (inferring): Actually accepting (= *explicitly* acknowledging commitment to accept) what one is *implicitly* committed to accept, in the sense of (a2).

This is **the *act* of *drawing* consequences**:

explicitly acknowledging some of the implications of one's other commitments.

It is a paradigmatic rational activity.

Conclusion:

If explicitation is inconsequential, then inference is impotent.

If inferring is not impotent, then inference can add new implicit commitments (consequences) and subtract old ones.

Inferring can be **ampliative** and lead to new knowledge (consequences), or **corrective** and guard us from (lead us to reject) old mistakes (implications).

Argument:

- Inference is *not* in general impotent, so explicitation is *not* in general inconsequential.
- So CM and CT are *no*t part of the universal and invariable structure of reason relations.
- So we want logical metavocabularies to be able to codify reason relations with *open* structure: *hyper*nonmonotonic (denies not just the stronger MO but also the weaker CM) and nontransitive (denies not just the stronger mixed-context Cut, but also the weaker shared-context Cut =CT).
- That is, the ideal logic should have the expressive power to make explicit reason relations that do *not* have the closure structure (monotonic and transitive) of classical (and intuitionist) *logical* reason relations.

Note: It need not be a nonmonotonic (and nontransitive) *logic*.
The *purely logical* consequence relation of NM-MS is fully monotonic and transitive.(!)
But the consequence and incompatibility relations of the *logically extended vocabulary* NM-MS defines are *not*—or it couldn't *conservatively* extend a substructural base vocabulary.
But it must be a logic with the expressive power explicitly to express reason relations that are both hypernonmonotonic and nontransitive.

2. A *vocabulary* $=_{\mathrm{df.}}$ a *lexicon*, L a set of sentences, together with a set R^2 of *reason relations* (sequents, codifying good implications) on that lexicon: $<L,R^2>$.
$R^2 \subseteq \mathscr{P}(L) x \mathscr{P}(L)$. $<\Gamma,\Delta> \in R^2$ iff $\Gamma |\sim \Delta$.

3. Logical vocabularies are *rational metavocabularies*, in the sense that the *lexicon* and *reason relations* of the *logical* metavocabulary is *wholly determined* in a systematic way by the lexicon and reason relations of some *base* vocabulary.
Further, logical vocabularies are distinguished from other rational metavocabularies (for instance, semantic ones), by being *conservative extensions* of their base vocabularies.
A logical vocabulary is a *conservatively extended elaboration* of the base vocabulary.

4. Logical vocabulary is a) *el*<u>*el*</u>*aborated* from and b) *ex*<u>*ex*</u>*plicative* of ("LX for") its base vocabulary.
Conditionals make *implications* explicit in the form of claimables expressed by sentences:
Deduction-Detachment (DD) Condition on Conditionals: $\Gamma |\sim A \rightarrow B$ iff $\Gamma, A |\sim B$.
Negation makes *incompatibilities* explicit in the form of claimables expressed by sentences:
Incoherence-Incompatibility (II) Condition on Negation: $\Gamma |\sim \neg A$ iff $\Gamma \# A$.

5. In addition, I articulated the expressive role characteristic of *logical* rational metavocabularies as being ***universally* and *comprehensively* LX**.

To say that they are *universally* LX is to say that they can be elaborated from and explicative of *any* and *all* vocabularies.
To say that they are *comprehensively* LX is to say that they explicate the reason relations not only of the *base* vocabulary, but also of the *logically extended* vocabulary that is LX for that base.

6. Conservatively extended elaboration of base *lexicon*:
L is the smallest (by inclusion) superset of L_0 (the base of the induction) such that if the elements above the line are in L, then so are the elements below the line (the inductive step):

$$\frac{\alpha, \beta \in L}{\neg\alpha \in L \quad \alpha \rightarrow \beta \in L \quad \alpha \& \beta \in L \qquad \alpha \vee \beta \in L.}$$

7. Conservatively extended elaboration of base *reason relations* (implications as sequents):
Corresponding to the requirement that $L_0 \subseteq L$, we have (base of the induction):

Axiom of NM-MS:

$$\frac{\Gamma \mid\!\sim_0 \Delta}{\Gamma \mid\!\sim \Delta}$$

And corresponding to the rules we close under (inductive step) are rules of the form:

L&: $$\frac{\Gamma, A, B \mid\!\sim \Theta}{\Gamma, A\&B \mid\!\sim \Theta}$$

R&: $$\frac{\Gamma \mid\!\sim A, \Theta \qquad \Gamma \mid\!\sim B, \Theta}{\Gamma \mid\!\sim A\&B, \Theta}$$

R^2 is then the smallest (by inclusion) superset of $R^2{}_0$ (that is the effect of the Axiom for NM-MS) such that if the sequents (implications, reason relations) above the line are in R^2, then so are the sequents below the line.

8. We conservatively extend the reason relations of the base vocabulary to reason relations for the logically extended vocabulary by following this model and, in the context of the Axiom for NM-MS (above) applying *all* the connective rules for
NM-MS (NonMonotonic-MultiSuccedent logic):

Expressive (Principal) **Connectives:**

L→: $\dfrac{\Gamma \mid\!\sim \Theta, A \qquad B, \Gamma \mid\!\sim \Theta}{A \rightarrow B, \Gamma \mid\!\sim \Theta}$

R→: $\dfrac{A, \Gamma \mid\!\sim \Theta, B}{\Gamma \mid\!\sim \Theta, A \rightarrow B}$

L¬: $\dfrac{\Gamma \mid\!\sim A, \Theta}{\neg A, \Gamma \mid\!\sim \Theta}$

R¬: $\dfrac{A, \Gamma \mid\!\sim \Theta}{\Gamma \mid\!\sim \neg A, \Theta}$

Aggregative (Auxiliary) **Connectives:**

L&: $\dfrac{\Gamma, A, B \mid\!\sim \Theta}{\Gamma, A \& B \mid\!\sim \Theta}$

R&: $\dfrac{\Gamma \mid\!\sim A, \Theta \qquad \Gamma \mid\!\sim B, \Theta}{\Gamma \mid\!\sim A \& B, \Theta}$

L∨: $\dfrac{A, \Gamma \mid\!\sim \Theta \qquad B, \Gamma \mid\!\sim \Theta}{A \vee B, \Gamma \mid\!\sim \Theta}$

R∨: $\dfrac{\Gamma \mid\!\sim A, B, \Theta}{\Gamma \mid\!\sim A \vee B, \Theta}$